2690 [April, 1918]. Proposed by E. V. HUNTINGTON, Harvard University.

Find the maximum value of

$$y = \frac{\sin x \cos (x + \varphi)}{\cos (x + \theta) \sin (x + \beta + \theta)}. \tag{1}$$

SOLUTION BY J. B. REYNOLDS, Lehigh University.

Taking the logarithm of both sides, differentiating, and setting the result equal to zero we get

$$\cot x - \tan (x + \varphi) + \tan (x + \theta) - \cot (x + \beta + \theta) = 0 \tag{2}$$

and from the two forms of this

$$\cot x - \cot (x + \beta + \theta) = \tan (x + \varphi) - \tan (x + \theta)$$

and

$$\cot x + \tan (x + \theta) = \cot (x + \beta + \theta) + \tan (x + \varphi)$$

we get

$$\frac{\sin (\beta + \theta)}{\sin (\varphi - \theta)} = \frac{\sin x \sin (x + \beta + \theta)}{\cos (x + \varphi) \cos (x + \theta)} \tag{3}$$

and

$$\frac{\cos (x + \varphi)}{\cos (x + \theta)} = \frac{\cos (\beta + \theta - \varphi)}{\cos \theta} \cdot \frac{\sin x}{\sin (x + \beta + \theta)}. \tag{4}$$

By (3) and (4), we have

$$\sqrt{\frac{\cos (\beta + \theta - \varphi) \sin (\varphi - \theta)}{\cos \theta \sin (\beta + \theta)}} = \frac{\cos (x + \varphi)}{\sin x} = \cot x \cos \varphi - \sin \varphi.$$

Hence,

$$\cot x = \tan \varphi + \sec \varphi \sqrt{\frac{\cos (\beta + \theta - \varphi) \sin (\varphi - \theta)}{\cos \theta \sin (\beta + \theta)}}. \tag{5}$$

(4) and (1) give, where y_1 is the maximum value of y,

$$y_1 = \frac{\cos (\beta + \theta - \varphi)}{\cos \theta} \cdot \frac{\sin^2 x}{\sin^2 (x + \beta + \theta)};$$

whence

$$\frac{1}{\sqrt{y_1}} = \sqrt{\frac{\cos \theta}{\cos (\beta + \theta - \varphi)}} \{\cos (\beta + \theta) + \sin (\beta + \theta) \cot x\}$$

and eliminating $\cot x$ by (5), we have

$$y_1 = \cos^2 \varphi / \{\sqrt{\cos \theta \cos (\beta + \theta - \varphi)} + \sqrt{\sin (\varphi - \theta) \sin (\beta + \theta)}\}^2.$$

As in 2689 we may prove that y_1 is a maximum as found for the case $\beta = \varphi$ from which we may infer from the manner that β enters into the expression that it is a maximum for all values of β.

2692 [April, 1918]. Proposed by J. L. RILEY, Stephenville, Texas.

A cube is cut at random by a plane, what is the chance that the section is a hexagon?

SOLUTION BY C. F. GUMMER, Queen's University, Kingston, Ont.

Let the faces of the cube be $x = \pm 1$, $y = \pm 1$, and $z = \pm 1$. Let the plane be

$$lx + my + nz = p,$$

where $n = \cos \theta > 0$, $l = \sin \theta \cos \phi$, $m = \sin \theta \sin \phi$. The normal from the origin is equally likely to have any direction, and, when the direction is fixed, equally likely to have any length. Since $\sin \theta d\theta d\phi$ is an element of the solid angle traced by the normal, and dp an element of its length, the relative probability of the plane lying within given limits is $\iiint \sin \theta dp d\theta d\phi$. On account of the symmetry of the cube, it may be assumed that the normal through O to the plane meets the surface of the cube within the triangle (0, 0, 1), (1, 0, 1), and (1, 1, 1); so that $n > l > m > 0$, that is, $0 < \phi < \pi/4$, $0 < \theta < \tan^{-1} \sec \phi$.